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SIMPLE METHOD OF USING ONBOARD OPTICAL MEASUREMENTS TO PREDICT THE ORBIT ESTABLISHED AFTER THE LUNAR-ASCENT-TRANSFER ORBIT

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16. Abstract

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SUMMARY

An analytical study has been made to develop a simple procedure for determining the apocynthion and pericynthion altitudes of the orbit established after ascent from the lunar surface. The procedure consisted of measuring the angle between the line of sight to the sun and that to the lunar horizon at two specified times on the established orbit and then predicting the altitudes graphically from these angular measurements. The effects of errors on the procedure were examined, and it appears that the technique would be useful for predicting the apocynthion and pericynthion altitudes of the established orbit.

INTRODUCTION

One phase of the Apollo lunar landing mission is the ascent of the lunar module or transfer vehicle from the lunar surface into an orbit around the moon for subsequent rendezvous with the command module, which is in a circular orbit about the moon at an altitude of about 80 nautical miles (150 000 meters). The primary guidance during the initial lunar missions is scheduled to be automatic. However, mission reliability and crew safety can be enhanced by development of simplified manual procedures that require a minimum of instrumentation to back up the automatic systems.

One approach for an emergency launch from the lunar surface is to use a simple, visually guided method, such as presented in reference 1, to inject the lunar module into a nominal Hohmann orbit with an apocynthion altitude of 80 nautical miles (150 000 meters). Near this apocynthion altitude, a modification to the orbit can be made that will result in an approximately circular orbit. This modification to the orbit is accomplished by orienting the thrust axis to a specified constant angle relative to the lunar horizon and then thrusting for a predetermined length of time; this constant-angle-thrusting maneuver will be referred to herein as the standard orbit modification.

The purpose of this study is to examine the orbit resulting from the standard orbit modification when the transfer vehicle was initially on an off-nominal transfer orbit, referred to as the established orbit herein, and to develop a simple procedure for determining the apocynthion and pericynthion altitudes of the established orbit in order to insure that the vehicle is in a safe orbit.

The procedure used in this study consisted of measuring the angle between the sun and lunar horizon at two specified times on the established orbit, and then using these angular measurements to determine the apocynthion and pericynthion altitudes of the orbit. The effects of errors in injection conditions, standard orbit modification, sighting angles, and sighting times are considered.

SYMBOLS

The units used for the physical quantities defined in this paper are given both in the U.S. Customary Units and the International System of Units (SI). Factors relating the two systems are given in reference 2.

B angle between sun and lunar horizon at second visual sighting, measured from transfer vehicle, degrees

$D = \cos^{-1} \frac{r_m}{r_m + h}$, degrees

g_m gravitational acceleration at surface of moon, 5.32 feet/second^2 (1.62 meters/second^2)

h altitude above lunar surface, feet (meters)

h_a altitude at apocynthion, feet (meters)

h_p altitude at pericynthion, feet (meters)

K angle between sun and lunar horizon at first visual sighting, measured from transfer vehicle, degrees

r radial distance from center of moon, feet (meters)

r_m radius of moon, 5 702 000 feet (1 738 000 meters)

$r\dot{\theta}$ velocity component perpendicular to radius vector, feet/second (meters/second)

$\dot{r}$ velocity component along radius vector, feet/second (meters/second)

S angular location of sun above launch-site horizontal, degrees

t time from initial transfer-orbit injection, seconds

t_B time of second visual sighting, seconds

t_K time of first visual sighting, seconds

V_i impulsive-velocity change for standard orbit modification, feet/second (meters/second)

δ angle between local horizontal at t_K and line parallel to launch-site horizontal, degrees

ζ angle between thrust axis and lunar horizon in direction of motion, degrees or radians

θ angular travel over lunar surface, degrees or radians

θ_L angular location of injection point from launch site, degrees

λ angular location from launch site when second angular measurement is made, degrees

Subscripts:

o initial injection conditions of transfer orbit

V standard oribt modification

1 transfer orbit

2 established orbit

Dots over symbols indicate derivatives with respect to time. A Δ preceding a parameter indicates a change in that parameter from its nominal value.

STATEMENT OF PROBLEM

The purpose of this study is to examine a simple procedure for determining the apocynthion and pericynthion altitudes of the orbit resulting from the standard orbit modification when the transfer vehicle is initially on an off-nominal transfer orbit. It is assumed that a transfer vehicle has been launched from the lunar surface and injected into a transfer orbit. This nominal orbit is a Hohmann transfer orbit, which has an apocynthion of 80 nautical miles (150 000 meters), as shown in figure 1. Near apocynthion of the nominal transfer orbit, a standard orbit modification is made which will place the transfer vehicle in approximately a circular orbit at 80 nautical miles (150 000 meters) above the lunar surface. If the transfer vehicle is not on the nominal transfer orbit and if the same standard orbit modification is applied, a different orbit will be established. Figure 2 is an illustration of the orbit established when, for example, the transfer orbit was low (apocynthion below 80 nautical miles (150 000 meters)) prior to application of the standard orbit modification.

The procedure examined consists of measuring the angle between the line of sight to the sun and that to the lunar horizon at two specified times on the established orbit, as shown in figure 3, and then determining the apocynthion and pericynthion altitudes of the established orbit from these angular measurements. This technique is simple and requires only a sighting device (such as a sextant), a time reference, and previously prepared graphs.

ANALYSIS

The approach used in this paper was to compute a series of transfer orbits which had different apocynthion altitudes as a result of velocity injection errors, perform the standard orbit modification at the same fixed time for all the transfer orbits, and then examine the sighting angles at two specified times in the established orbit. Appreciable differences in the angles for the various trajectories would indicate that the sighting angles could be used to distinguish the orbits.

Transfer Orbit

At the end of the initial powered ascent which covers a range angle of $\theta_L = 10^{\circ}$ and terminates at an altitude of 50 000 feet (15 240 meters), the transfer vehicle is injected into the transfer orbit. Some characteristics of the nominal Hohmann transfer orbit are

$$h_o = 50\ 000 \text{ feet} \quad (15\ 240 \text{ meters})$$

$$\dot{r}_o = 0$$

$$(r\dot{\theta})_o = 5583 \text{ ft/sec} \quad (1702 \text{ m/sec})$$

$$h_{p,1} = 50\ 000 \text{ feet} \quad (15\ 240 \text{ meters})$$

$$h_{a,1} = 80 \text{ nautical miles} \quad (150\ 000 \text{ meters})$$

The various transfer orbits examined in this investigation had injection-velocity components which were varied simultaneously from the nominal values by approximately $-65 < \Delta(r\dot{\theta})_o < 45$ ft/sec $(-19.8 < \Delta(r\dot{\theta})_o < 13.7$ m/sec) and $|\Delta\dot{r}_o| \leqq 10$ ft/sec (3 m/sec). The values of $\Delta\dot{r}_o$ which can be found in reference 3 are larger than the values used in the present study. However, if larger values are considered, the general technique will essentially be the same, except for adjustments in the standard orbit modification and sighting times. The adjustments would be required because of the larger rotations of the transfer orbit due to larger values of $\Delta\dot{r}_o$. The deviations in $\dot{r}_o$ from zero caused the line of apsides to rotate a maximum of 8^o, whereas the apocynthion and pericynthion altitudes remained essentially unchanged. A positive value of $\Delta\dot{r}_o$ produces a rotation opposite to the direction of motion; a negative value produces a rotation in the direction of motion. The primary effect of the deviations in $(r\dot{\theta})_o$ from the nominal value was to vary the apocynthion altitudes of the transfer orbits between 200 000 and 700 000 feet (60 960 and 213 360 meters).

Standard Orbit Modification

The standard orbit modification was the same for all transfer orbits and consisted of orienting the thrust axis of the transfer vehicle to $\zeta_V = 22.9^o$ above the lunar horizon in the direction of motion and adding an impulsive velocity of 97.4 ft/sec (29.7 m/sec). A fixed time from injection t_V of 3600 seconds was selected for applying the standard orbit modification for all orbits. (See fig. 2.)

The length of time the horizon will be visible prior to the time that the standard orbit modification is made will be a function of the sun angle S at launch. As S increases, this viewing time will decrease. For the selected time of $t_V = 3600$ seconds, it was found that the time that the horizon was visible was at least 134 seconds for sun angles as large as 30^o at launch. This length of time was accepted as being sufficient for the astronaut to prepare for the standard orbit modification.

Fixed Sighting Times

The first fixed sighting time t_K was chosen to allow ample time for the astronaut to prepare for the first angular measurement after the standard orbit modification was made; the second sighting time t_B was chosen so that the second angular measurement occurred on the sunlit side of the moon. In addition, the two angular measurements were separated sufficiently in order to decrease the sensitivity of the predicted altitudes to the various possible errors. Under these guidelines, the time selected for the first angular measurement t_K was 300 seconds after the standard orbit modification $(t_K = 3900$ seconds$)$; and the time selected for the second angular measurement t_B was 1700 seconds after the standard orbit modification $(t_B = 5300$ seconds$)$. Figure 4 shows the angular location λ of the transfer vehicle from the launch site when the second angular measurement is made. The location of the transfer vehicle on the approach side of the launch site varies from approximately $\lambda = -68.5^o$ to $\lambda = -89.5^o$ for the range of $\Delta(r\dot{\theta})_o$ values used. The angle λ is rather insensitive to radial injection velocity error $\Delta\dot{r}_o$.

Equations of Motion

The equations of motion used were

$$\ddot{r} - r\dot{\theta}^2 + g_m\left(\frac{r_m}{r}\right)^2 = 0 \tag{1}$$

and

$$r\ddot{\theta} + 2\dot{\theta}\dot{r} = 0 \tag{2}$$

These equations describe planar movement of a point mass near a spherical homogeneous moon. By using the initial injection conditions h_o, $\dot{r}_o$, and $(r\dot{\theta})_o$, these equations were integrated to $t_V = 3600$ seconds where the standard orbit modification was applied. The standard orbit modification injected the transfer vehicle into the established orbit.

The sighting angles K and B were determined at specified times on this established orbit. The first sighting angle K measured relative to the horizon in the direction of motion can be expressed in terms of the angles θ and D, which occur at the time t_K by the equation

$$K = D + \theta + \theta_L - S - 180^o \tag{3}$$

The geometric relationship may be obtained from figure 5. The second sighting angle B measured relative to the horizon opposite the direction of motion can be expressed similarly in terms of the angles θ and D, which occur at the time t_B by the equation

$$B = D - \theta - \theta_L + S + 360^o \tag{4}$$

The positions of the transfer vehicle at t_K and t_B, which are necessary to define the angles θ and D in equations (3) and (4), were obtained by integration of the equations of motion (1) and (2) after the standard orbit modification was made.

RESULTS AND DISCUSSION

Determination of Apocynthion and Pericynthion Altitudes of Established Orbit

If the standard orbit modification were applied exactly at apocynthion of the nominal transfer orbit, then, by definition, the resulting established orbit would be circular at 80 nautical miles (150 000 meters) above the lunar surface; however, the standard orbit modification has been delayed a few seconds in this study; therefore, the resulting orbit is not circular. The data in figure 6 show the variation of the apocynthion and pericynthion altitudes of the established orbits as functions of the injection velocity errors $(\Delta r\dot{\theta})_o$ and $\Delta\dot{r}_o$. For no errors in the nominal injection-velocity components ($\Delta\dot{r}_o = 0$ and $\Delta(r\dot{\theta})_o = 0$), the nominal values of $h_{a,2}$ and $h_{p,2}$ differ by approximately 46 000 feet (14 021 meters). Figure 6 also shows that the standard orbit modification always results in safe orbits with minimum altitudes of about 33 nautical miles (61 000 meters) for the range of errors considered.

In addition to causing variations in $h_{a,2}$ and $h_{p,2}$, $\Delta(r\dot{\theta})_o$ and $\Delta\dot{r}_o$ produce changes in the sighting angles K and B. Figure 7 shows the relationships between the sighting parameters $K + S$ and $B - S$ and the injection velocity errors $\Delta(r\dot{\theta})_o$ and $\Delta\dot{r}_o$.

The results of figures 6 and 7 can be combined, as in figure 8, to relate $h_{a,2}$ and $h_{p,2}$ to the sighting parameters $K + S$ and $B - S$. If the sun angle S is assumed to be 30^o at launch and the angles K and B are found to be 23.18^o and 131.6^o, respectively, at the specified sighting times after the standard orbit modification is applied, from figure 8 the apocynthion and pericynthion of the established orbit are found to be approximately 500 000 feet (152 400 meters) and 440 000 feet (134 112 meters), respectively.

Error Analysis

The data on figure 8 present values of $h_{a,2}$ and $h_{p,2}$ that are exact for initial injection velocity errors ($\Delta\dot{r}_o$ and $\Delta(r\dot{\theta})_o$) within the ranges of this study. It was of interest to examine the accuracy in predicting $h_{a,2}$ and $h_{p,2}$ from figure 8 under the influence of various other errors. Errors considered were those associated with sighting angles, sighting times, injection altitude, and standard orbit modification.

In making the angular measurements, the astronaut should be consistent in using the same reference point on the sun. The actual measurement errors in sighting angles and sighting times could be reduced by making a number of sightings in the vicinity of the selected sighting times and then graphically obtaining the most likely value of the sighting angle. The effects of errors due to uncertainties in the lunar horizon, in ground range traveled during the powered ascent, and in the sun angle measured at launch can be considered as contributing factors to the overall sighting errors. For example, calculations show that an uncertainty of ±1 nautical mile (±2000 meters) in altitude of the lunar horizon produces maximum errors of about 0.2° in ζ_V, K, and B. Errors in S and θ_L produce direct equivalent errors in the sighting angles K and B as shown by equations (3) and (4), respectively; however, these errors should be small since S will probably be accurately determined while the vehicle is on the lunar surface and because reasonable errors in thrust cut-off time (0.2 second) result in only about 0.01° error in θ_L. As mentioned previously, these errors are considered to be incorporated in the total angular-measurement errors.

Individual errors.- The effects of individual errors in the sighting angles (K and B), sighting times $(t_K$ and $t_B)$, injection altitude (h_o), and standard-orbit-modification parameters $(\zeta_V$, t_V, and $V_i)$ on the predicted values of $h_{a,2}$ and $h_{p,2}$ were computed. Each individual error was examined for nine combinations of $\Delta(r\dot{\theta})_o$ and $\Delta\dot{r}_o$. For these combinations $\Delta(r\dot{\theta})_o$ was permitted to be either -65, 0, or 45 ft/sec (-19.8, 0, or 13.7 m/sec), whereas $\Delta\dot{r}_o$ could be either -10, 0, or 10 ft/sec (-3, 0, or 3 m/sec). Table 1 shows the maximum errors in the predicted values of $h_{a,2}$ and $h_{p,2}$ that were obtained for the various individual errors examined in this study. The most influential individual errors are K, B, V_i, and h_o, and the magnitude of the largest error in the predicted values of $h_{a,2}$ or $h_{p,2}$ is less than 4 nautical miles (7000 meters). Thus, only the four most influential errors were considered for the combination error analysis. Combination errors of $\pm 1^{\circ}$ in the sighting angles, ±1 nautical mile (±2000 meters) in injection altitude, and ±3 ft/sec (±0.9 m/sec) in impulsive velocity were examined.

Combination errors.- All possible combinations of the discrete-error values shown in table 1 for the most influential errors K, B, V_i, and h_o were examined over the previously mentioned nine combinations of $\Delta(r\dot{\theta})_o$ and $\Delta\dot{r}_o$. These conditions resulted in 729 different cases of prediction errors in $h_{a,2}$ and $h_{p,2}$. It was found that the errors in the predicted values of $h_{a,2}$ and $h_{p,2}$ were within the following boundaries:

$$-34\,595 \text{ feet } (-10\,545 \text{ meters}) \leqq \text{error in } h_{a,2} \leqq 36\,162 \text{ feet } (11\,022 \text{ meters})$$

$$-43\,271 \text{ feet } (-13\,189 \text{ meters}) \leqq \text{error in } h_{p,2} \leqq 40\,101 \text{ feet } (12\,223 \text{ meters})$$

As indicated by these boundaries, the largest errors in the predicted values of $h_{a,2}$ and $h_{p,2}$ were approximately ±7.0 nautical miles (13 000 meters).

Out-of-plane considerations.- The graphs of this study were prepared under the assumption that the sun was located in the plane of motion; however, similar graphs could be prepared for specified sun locations out of the plane of motion. It was of interest to examine how the sighting angles of this study are affected by out-of-plane sun location. Calculations showed that for sun locations of 10° out of the plane of motion, the maximum error introduced into the sighting angles was about 0.2°. When V_i is applied slightly out of the plane of motion, the desired resultant velocity does not vary appreciably. For example, calculations show that an out-of-plane application of V_i at 5° to the nominal plane will only change the resultant velocity by approximately 1 ft/sec (0.3 m/sec) for this study. The variation in K and B, which are measured after V_i is applied at 5° to the nominal plane, is small since this maneuver only changes the plane of motion of the transfer vehicle at V_i application by approximately 0.02°.

CONCLUDING REMARKS

An analytical study was made to develop a simple procedure which used a minimum of instrumentation to determine the apocynthion and pericynthion altitudes of the orbit established after ascent from the lunar surface. The procedure developed consisted of measuring the angle between the lines of sight to the sun and lunar horizon at two specified times on the established orbit and then predicting the apocynthion and pericynthion altitudes from these angular measurements. The effects of errors in sighting angles, sighting times, injection conditions, and standard orbit modification were considered.

The errors in the predicted altitudes were found to be most sensitive to errors in sighting angles, injection altitude, and impulsive velocity required for the standard orbit modification. Combination errors of $\pm 1^{\circ}$ in the sighting angles, ± 1 nautical mile (± 2000 meters) in injection altitude, and ± 3 ft/sec (± 0.9 m/sec) in impulsive velocity were examined. It was found that the apocynthion and pericynthion altitudes could be determined to within about ± 7.0 nautical miles (± 13000 meters) of their actual values. Preliminary examination of out-of-plane errors indicated that small out-of-plane errors can be tolerated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., November 6, 1969.

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TABLE 1.- EFFECT OF INDIVIDUAL ERRORS

Individual error in –	Discrete errors considered	Maximum magnitude of errors in predicted values of –			
		$h_{p,2}$		$h_{a,2}$	
		ft	meters	ft	meters
K	0, $\pm 1^{\circ}$	22 447	6842	20 759	6327
B	0, $\pm 1^{\circ}$	20 149	6141	19 445	5927
V_i	0, ±3 ft/sec (±0.9 m/sec)	14 123	4305	13 650	4161
h_o	0, ±1 n. mi. (±2000 m)	7 027	2142	6 048	1843
t_K	0, ±1 sec	2 088	636	1 336	407
t_B	0, ±1 sec	2 929	893	2 177	664
t_V	0, ±1 sec	2 217	676	1 393	425
ζ_V	0, $\pm 1^{\circ}$	3 295	1004	2 642	805

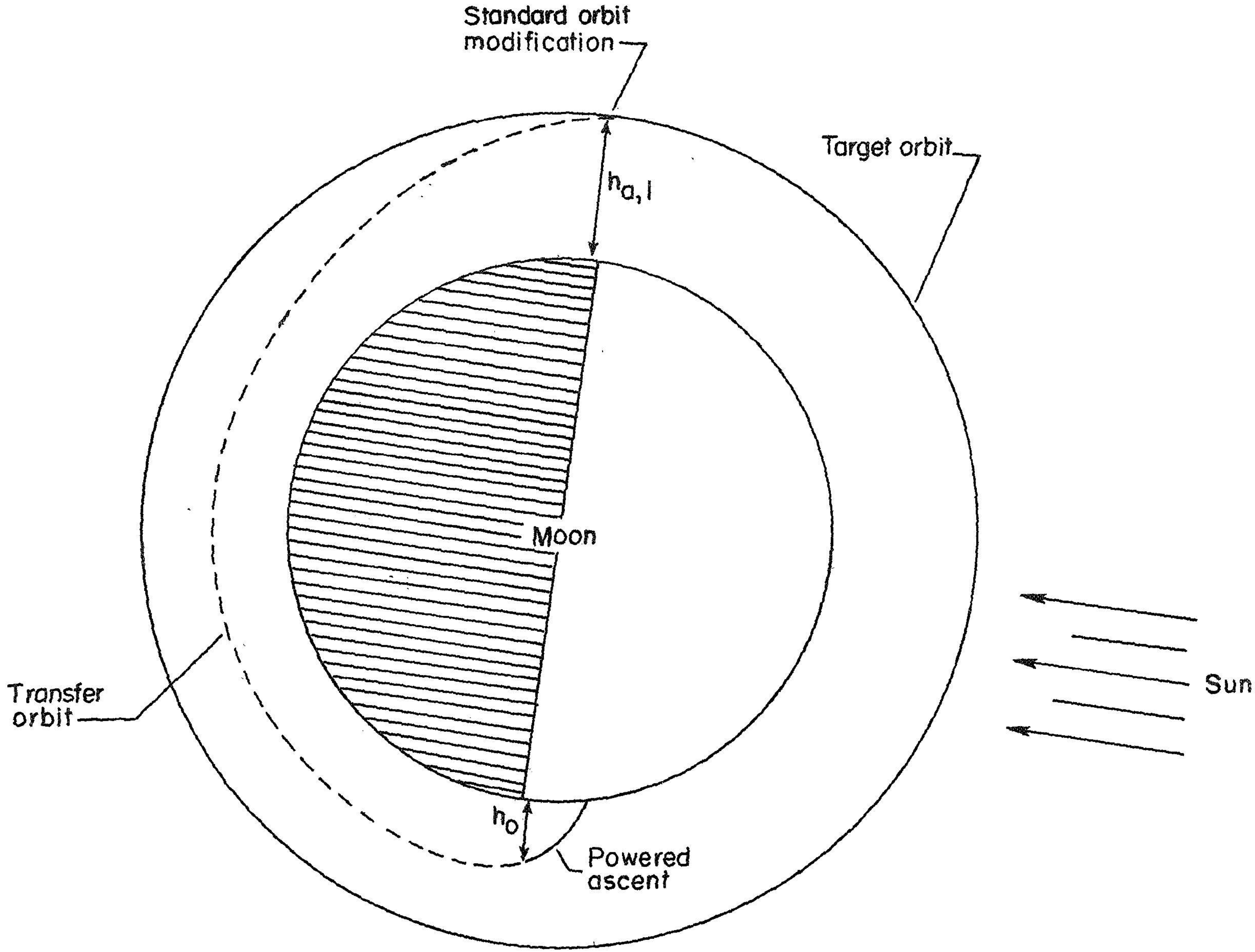


Figure 1.- Illustration of nominal transfer orbit.

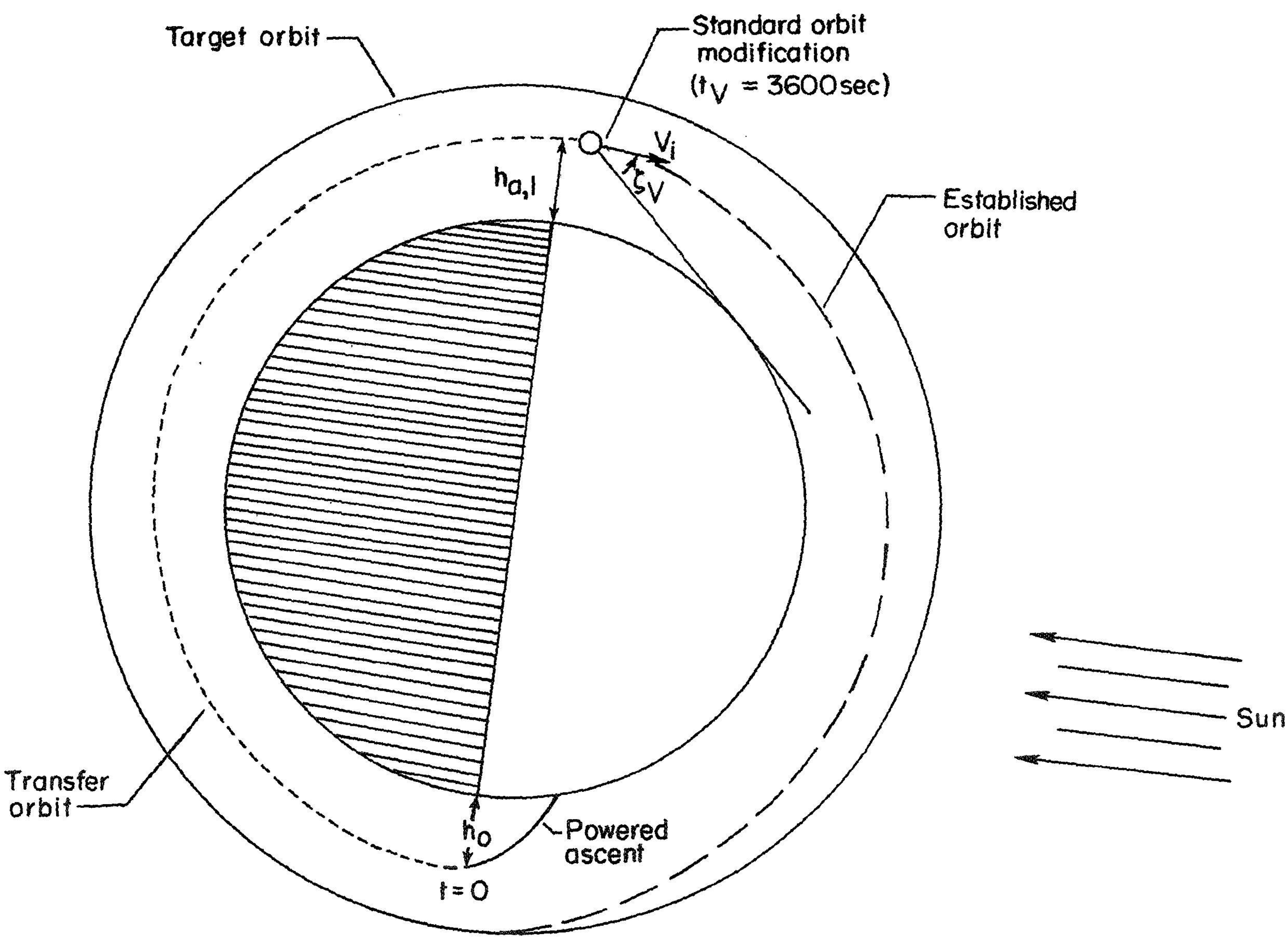


Figure 2.- Illustration of transfer orbit, standard orbit modification, and established orbit.

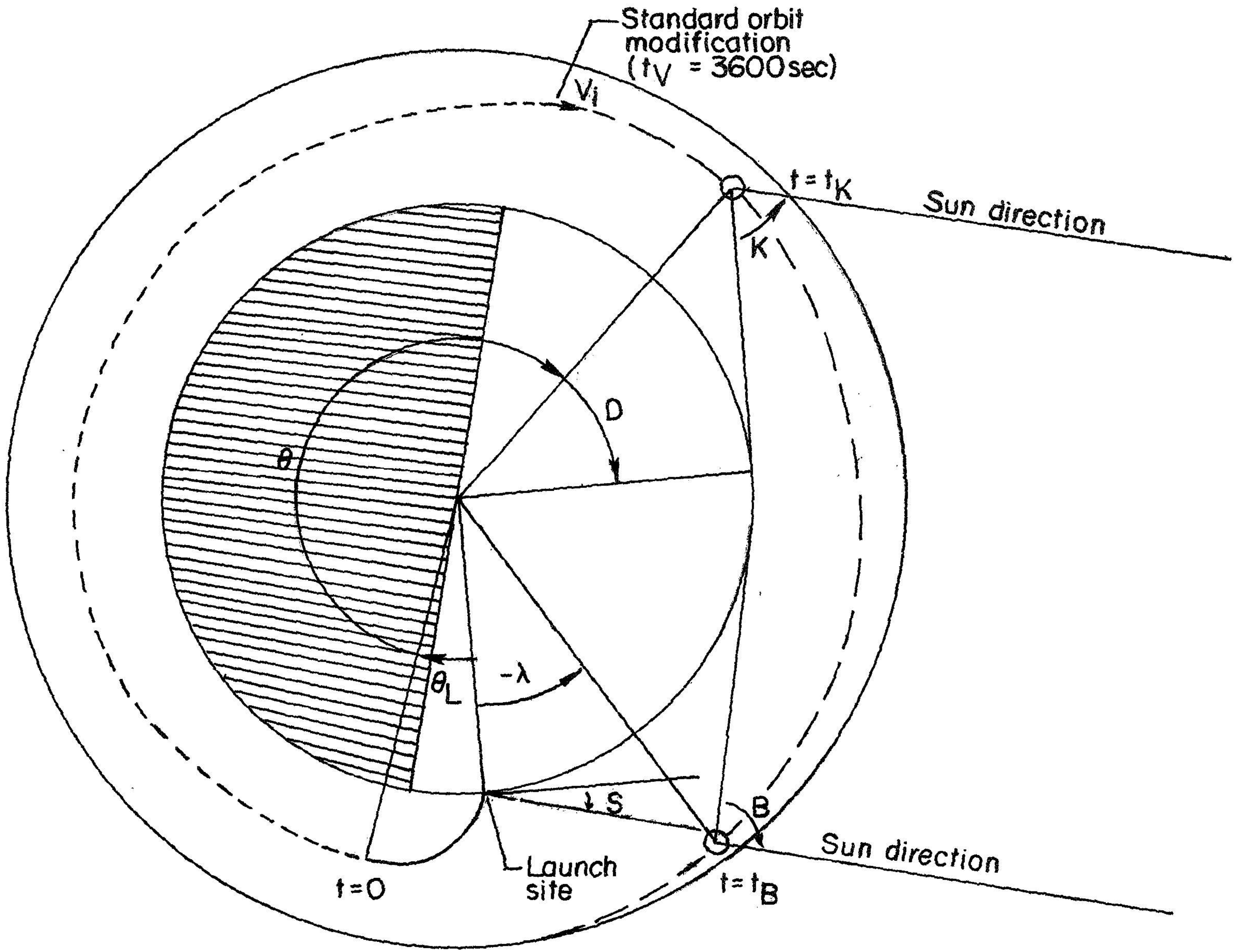


Figure 3.- Illustration of sighting angles used in orbit determination.

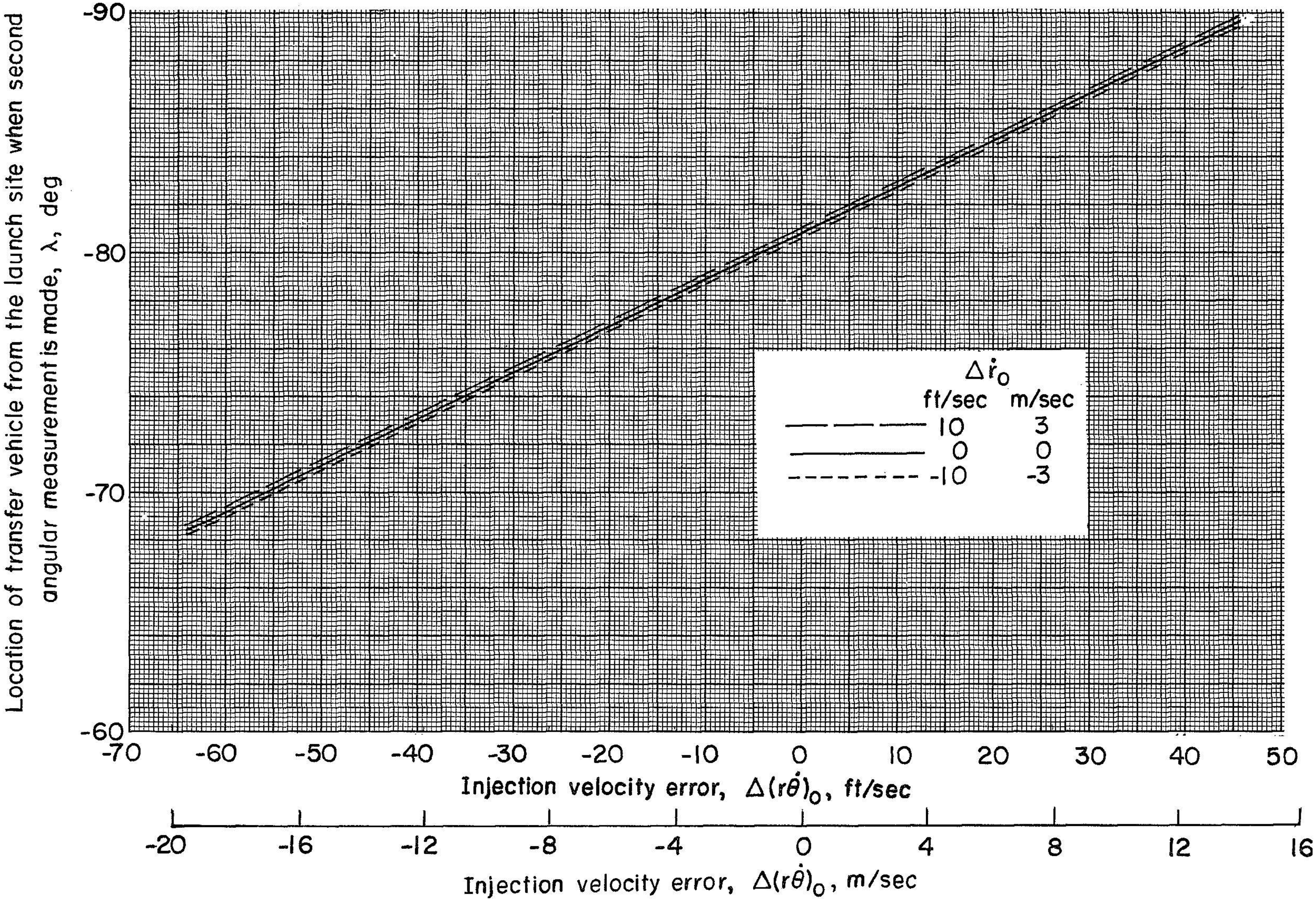


Figure 4.- Angular travel from launch site to second-angular-measurement location B.

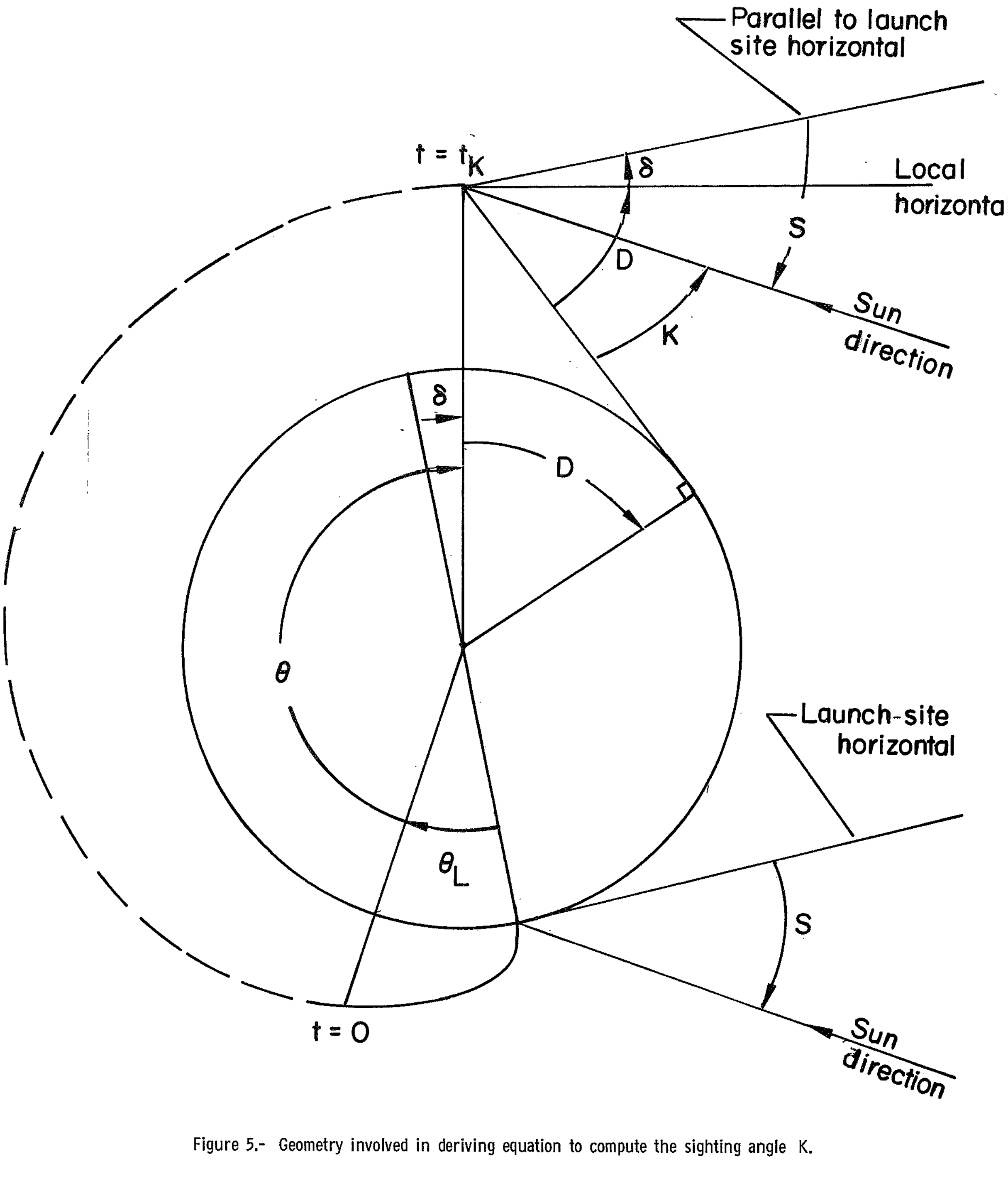


Figure 5.- Geometry involved in deriving equation to compute the sighting angle K.

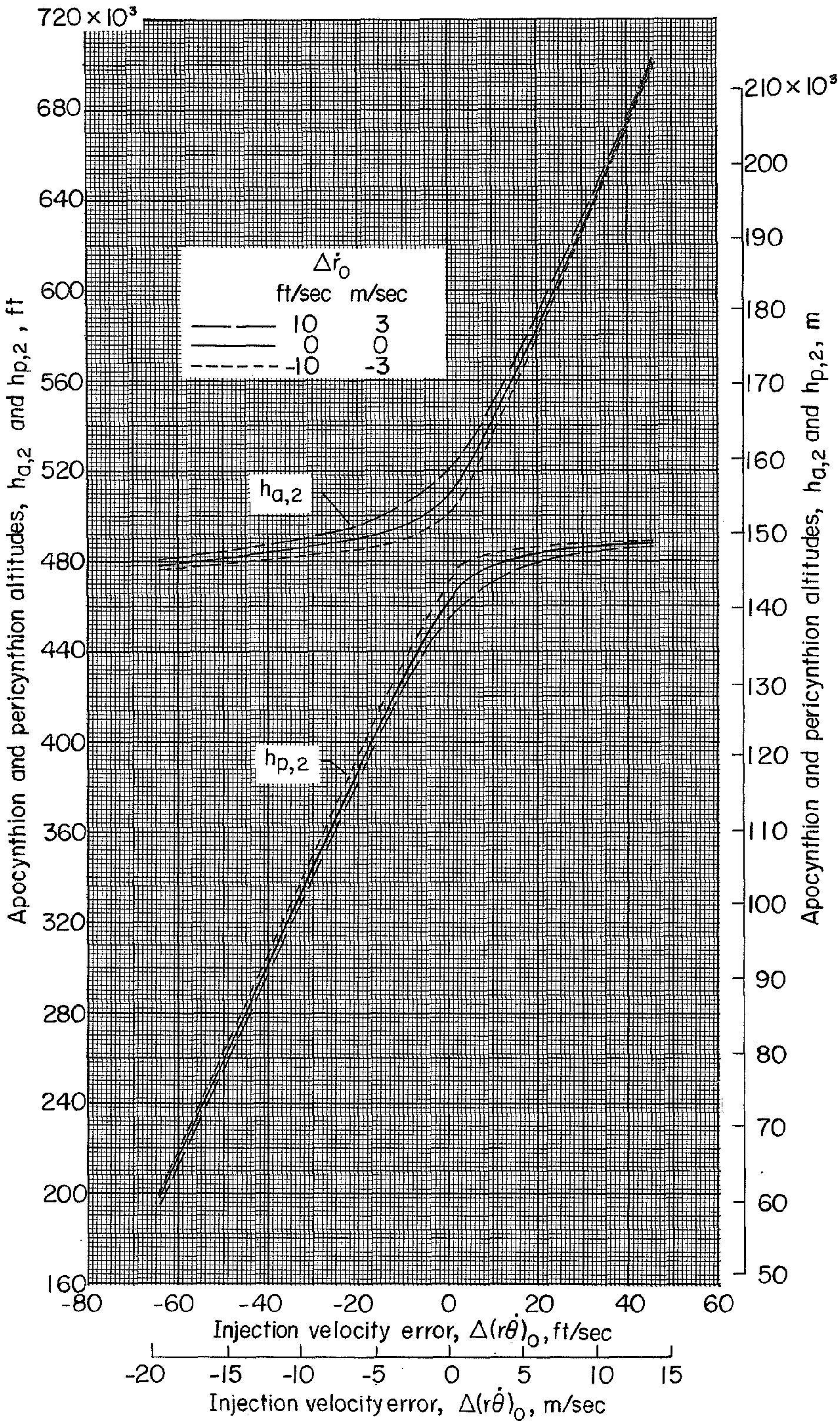


Figure 6.- Apocynthion and pericynthion altitudes of the established orbit as functions of injection velocity errors.

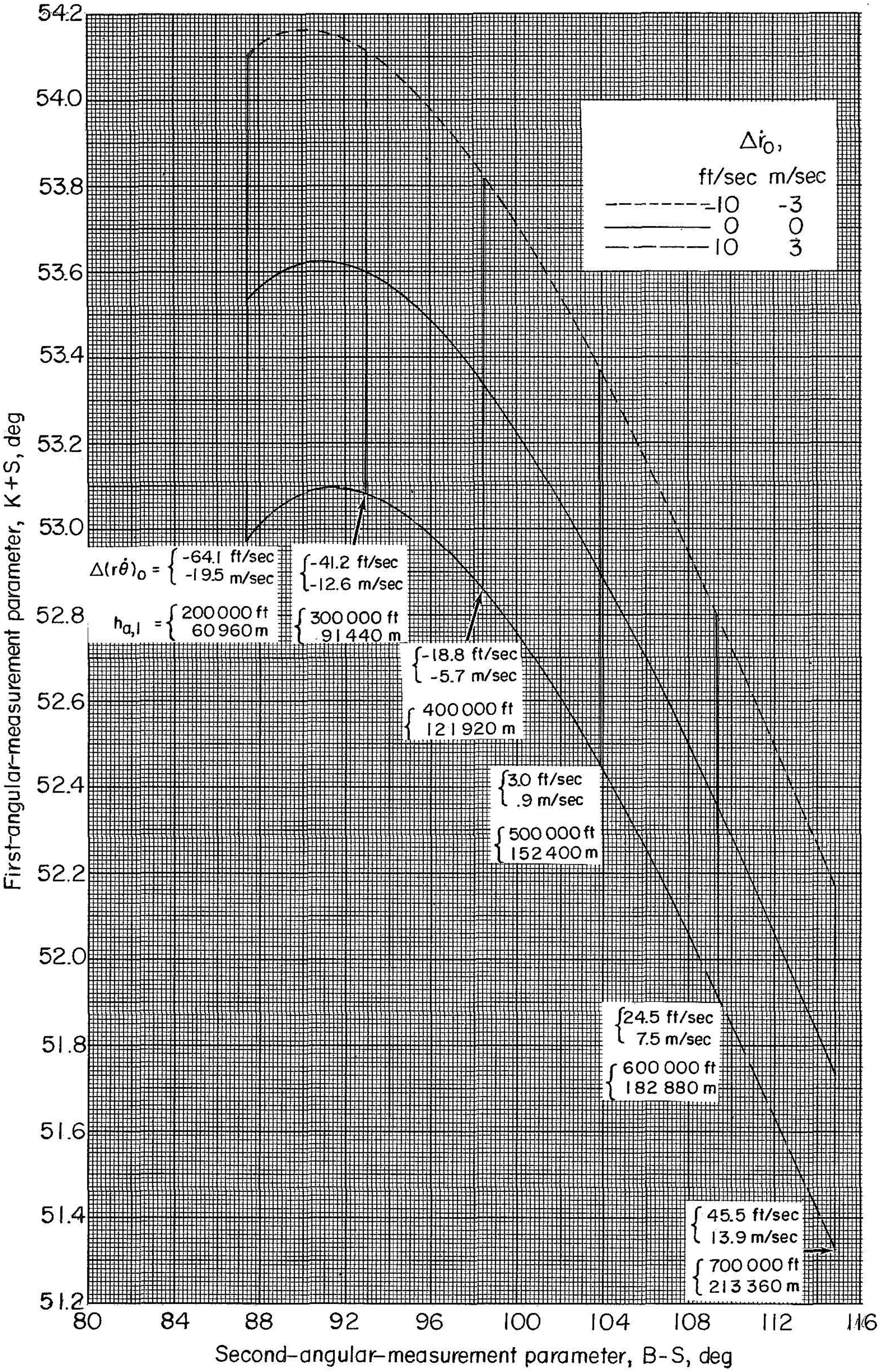
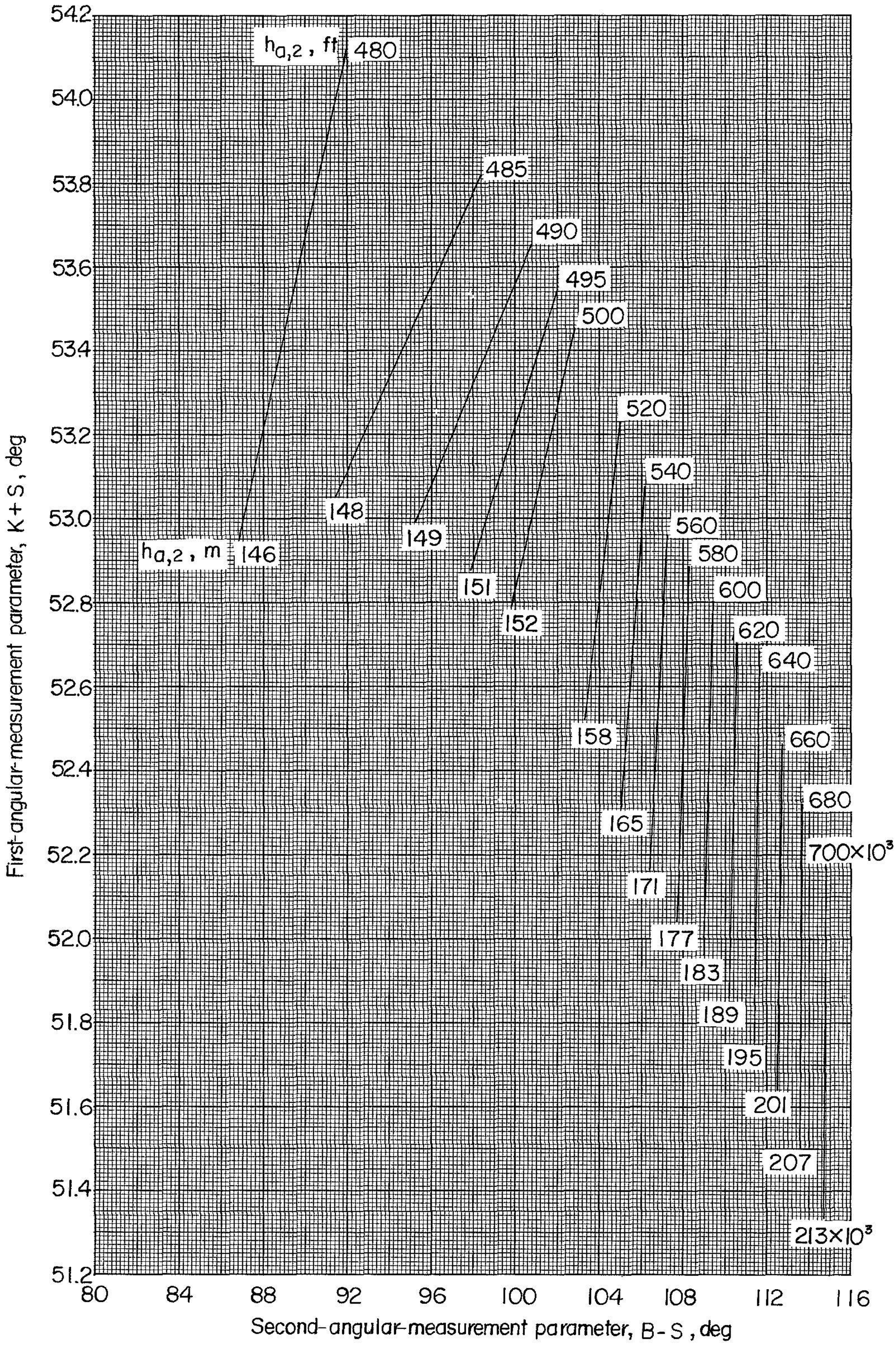
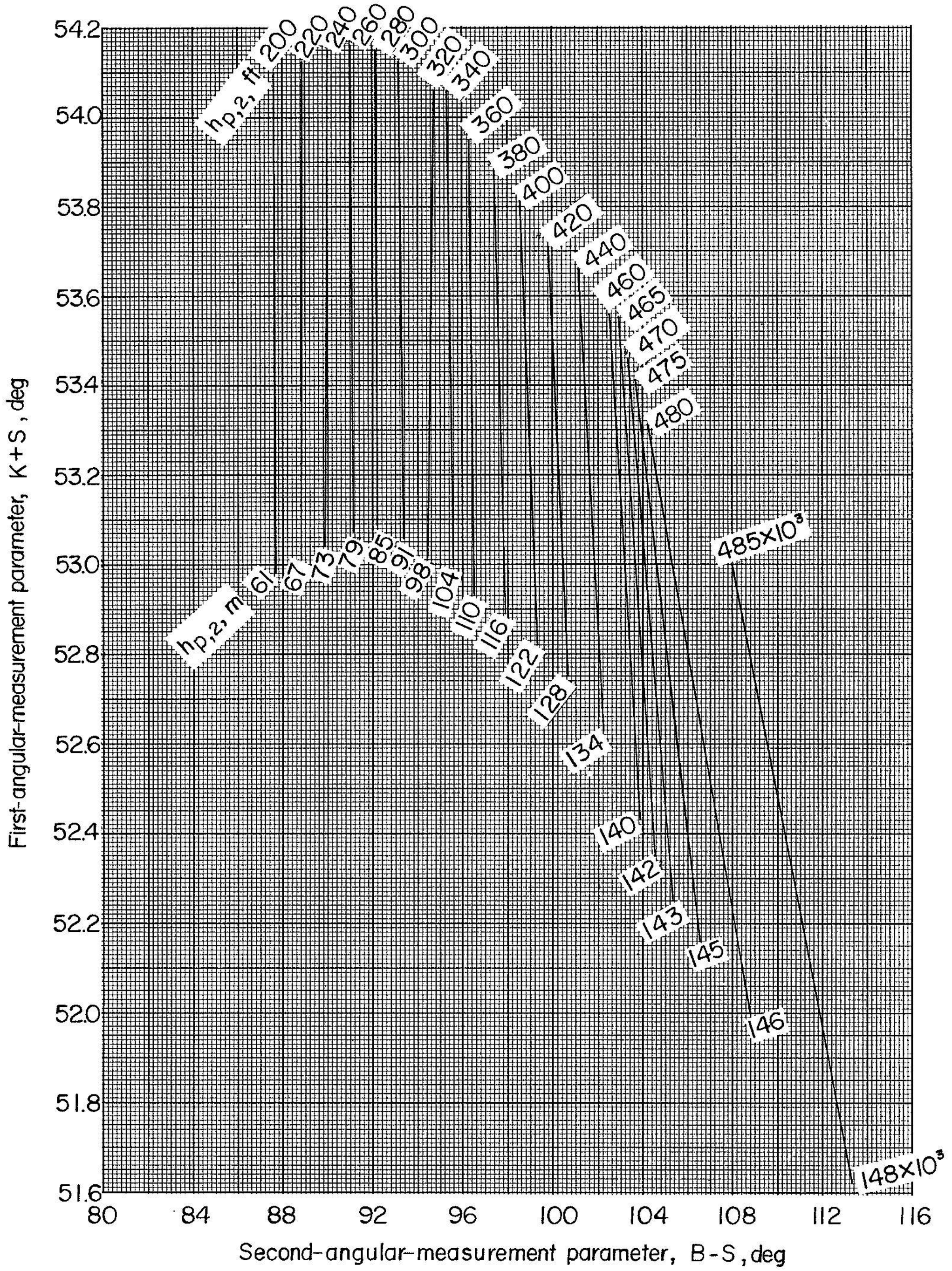


Figure 7.- Injection velocity errors as functions of sighting parameters.



(a) $h_{a,2}$.

Figure 8.- Apocynthion and pericynthion altitudes of the established orbit as functions of the sighting parameters.



(b) $h_{p,2}$.

Figure 8.- Concluded.

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